Capacitors Comparator using Astable Multivibrator with variable Duty Cycle

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Abstract

This project deals with developing a circuit that can be used as a capacitors comparator. Popular methods such as AC bridges and digital instruments for capacitor measurements are described. Main focus is on developing a simple working model to differentiate between capacitors based on time constant when capacitors are in series with resistors. AC bridges require AC source and manual control, our design does not have these requirements. Digital instruments are complex and expensive, on the contrary components used in this paper are very cheap. Duty cycles of outputs of Astable Multivibrator will be different based on which capacitor is larger. We compare duty cycles with low-pass filters and eventually feed the signals into a comparator to produce one or zero as indication of larger or smaller capacitor. Two important aspects of our design: sensitivity and response time, are discussed. Software simulation and experiments are preformed to confirm the performance of proposed circuit. Applications may include any capacitive transducer to monitor some physical quantity.

1 Introduction

Comparison of capacitance values is often needed in industrial and process control systems and instrumentation. For instance, certain real-world objects poses variable capacitance value as a sign of their certain properties. Comparison with reference capacitance might be required to trigger an alarm or to obtain useful information in control systems. Historically bridge based capacitor comparators have been known. Variations on the Wheatstone bridge can be used to measure capacitance, inductance, impedance and other quantities. Usage of these bridges results in very precise and most sensitive measurements of capacitors. Although simple, this method requires AC current and complex elements such as RMS galvanometer, variable resistances or variable capacitances. Purpose of our project is to describe means for comparing capacitance values of capacitors electronically. Designed circuits must be simple but still providing reasonable precision and fast response.

1.1 Background

There are several significant methods for comparing and measuring capacitors. Digital instruments usually measure the period of vibrations of circuit cointaining the unknown capacitor, or measuring a time reguired for a capacitor to charge up to a certain value, provided that constant current is flowing into a capacitor [1]. AC source with variation of Wheatstone bridge circuit is often used in comparing and measuring capacitors [2].

1.1.1 Digital instruments

Since capacitance is linearly proportional to the time constant, when a capacitor is charged by a constant current cource and discharged through a fixed resistance, we can use a 555 timer along with some digital test equipment to measure capacitances.

One obvious way is to measure the time period of oscillations. By choosing the right size of charging resistance, we can get a reading directly in microfarads or nanofarads. Unlike many capacitance measuring schemes, this one easily handles electrolytics up to a tens of thousands of microfarads.

A better way is to measure only the capacitor discharge time, as shown in Figure 1. With this method, any leakage in the capacitor under test will make the capacitor appear smaller in the value than it actually is, and is an effective indicator of how the test capacitor will behave in most timing and bypass circuits.

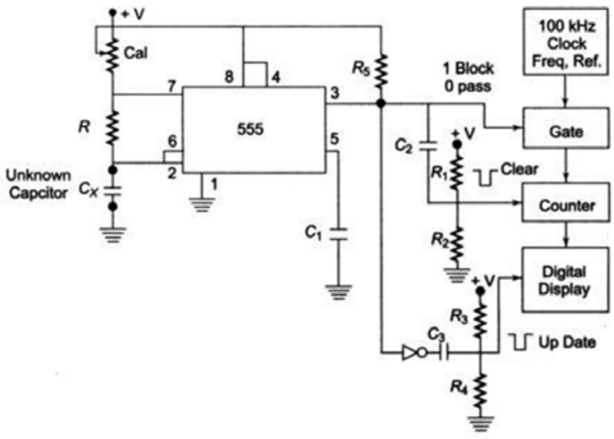

Figure 1: Block Diagram of a Basic Digital Capacitance Meter

In this circuit, the 555 timer is used as an astable multivibrator. At the peak of the charging curve, a digital counter is reset and a clock of 100kHz pulses is turned on and routed to the counter. When the discharge portion of the cycle is completed, the display is updated and the value of the capacitor is readout. By selecting the proper reference frequency and charging currents, one can obtain a direct digital display of the value of the capacitance.

It is important to properly shield the leads and keep them short for low capacity measurements, since the 50 Hz hum can cause some slight instability.[1]

1.1.2 Measurements using bridges

Bridge circuits are extensively used for measuring component values such as R. L and C. Since the bridge circuit merely compares the value of an unknown component with that of an accurately known component (a standard), its measurement accuracy can be very high. This is because the readout of this comparison is based on the null indication at bridge balance, and is essentially independent of the characteristics of the null detector. The measurement accuracy is therefore directly related to the accuracy of the bridge component and not to that of the null indicator used.

Series-Resistance-Capacitance Bridge

Figure 2 is a series-resistance-capacitance (RC) bridge, which is used for the comparison of a known capacitance with an unknown capacitance. The unknown capacitance is represented by C_x and R_x. A standard adjustable resistance R_1 is connected in series with a standard capacitor C_1. The voltage

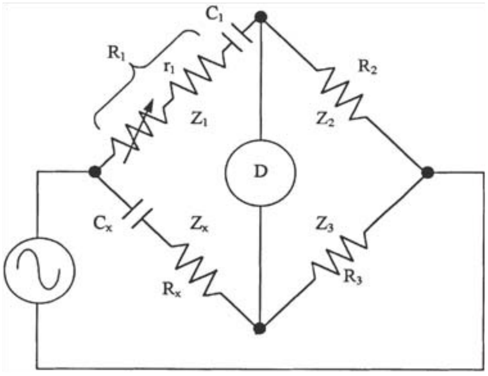

Figure 2: A series RC bridge. In these bridges, the unknown capacitance is compared with a known capacitance.

drop across R_1balances the resistive voltage drop when the bridge is balanced. The additional resistor in series with C_x increases the total resistive component, so that small values of R_1 will not be required to achieve balance. Generally, the bridge balance is most easily achieved when capacitive branches have substantial resistive components. To obtain balance, R_1 and either R_3 or R_4 are adjusted alternately. This type of bridge is found to be most suitable for capacitors with a high-resistance dielectric and hence very low leakage currents.[2]

At balance

$$Z_1 Z_3 = Z_2 Z_x \tag{1}$$

Substituting impedance values gives

$$\left(R_1 - \frac{j}{\omega C_1}\right) R_3 = \left(R_x - \frac{j}{\omega C_2}\right) R_2 \tag{2}$$

Equating the *real* terms gives

$$R_x = \frac{R_1 R_3}{R_2} \tag{3}$$

and equating *imaginary* terms gives

$$C_x = \frac{C_1 R_2}{R_3} \tag{4}$$

An improved version of the series RC bridge is the substitution bridge, which is particularly useful to determine the values of capacitances at radio frequencies. In this case, a series-connected RC bridge is balanced by disconnecting the unknown capacitance C_x and resistance R_x, and replacing it by an adjustable standard capacitor C_s and adjustable resistor R_s. After having obtained the balance position, the unknown capacitance and resistance C_x an R_x are connected in parallel to the capacitor C_s. The capacitor C_s and resistor R_s are adjusted again for the rebalance of the bridge. The changes in the DCs and DRs lead to unknown values as:

$$C_x = \Delta C_s \qquad \text{and} \qquad R_x = \Delta R_s \left(\frac{C_{s1}}{C_x}\right)^2$$

where C_{s1} is the value of C_s in the initial balance condition.

The Parallel-Resistance-Capacitance Bridge

Figure 3 illustrates a parallel-resistance-capacitance bridge. In this case, the unknown capacitance is represented by its parallel equivalent circuit C_x in parallel with R_x. The Z_2 and Z_3 impedances are pure resistors with either or both being adjustable. The Z_1 is balanced by a standard capacitor C_1 in parallel with an adjustable resistor R_1. The bridge balance is achieved by adjustment of R_1 and either R_2 or R_3. The parallel-resistance-capacitance bridge is found to be most suitable for capacitors with a low-resistance dielectric, hence relatively high leakage currents.

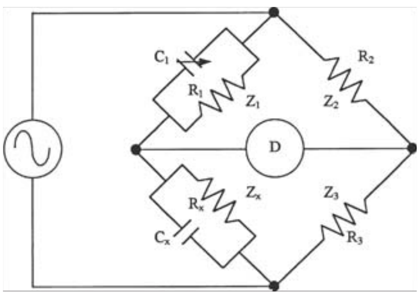

Figure 3: A parallel-resistance-capacitance bridge. The unknown capacitance is represented by its parallel equivalent circuit.

At balance:

$$\frac{1}{\left(\frac{1}{R_1} + j\omega C_1\right)} R_3 = \frac{1}{\left(\frac{1}{R_x} + j\omega C_x\right)} \tag{5}$$

Equating the real terms gives

$$R_x = \frac{R_1 R_3}{R_2} \tag{6}$$

and equating *imaginary* terms gives

$$C_x = \frac{C_1 R_2}{R_3} \tag{7}$$

The Wien Bridge

Figure 4 shows a Wien bridge. This is a special resistance-ratio bridge that permits two capacitances to be compared once all the resistances of the bridge are known.

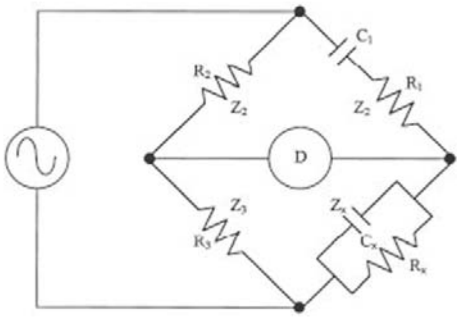

Figure 4: The Wien bridge. This bridge is used to compare two capacitors directly.

At balance, it can be proven that the unknown resistance and the capacitance are:

$$R_x = \frac{R_3 \left(1 + \omega^2 R_1^2 C_1^2\right)}{\omega^2 R_1 R_2 C_1^2} \tag{8}$$

and

$$C_x = \frac{C_1 R_2}{\left[R_3 \left(1 + \omega^2 R_1^2 C_1^2\right)\right]} \tag{9}$$

It can be also shown that:

$$\omega^2 = \frac{1}{R_1 C_1 R_x C_x} \tag{10}$$

As indicated in equation, the Wien bridge has an important application in determining the frequency in RC oscillators. In frequency meters, C_1 and C_x are made equal and the two capacitors are ganged together so that the frequency at which the null occurs varies linearly with capacitances.

1.2 Review

We have described several measuring and comparing devices for the capacitors. One might argue that these methods have some drawbacks. Balancing the bridge requires AC voltage source and the has to be operated manually, and therefore cannot be used in automated systems and are susceptible to gross reading errors. Digital measuring devices are better in terms of automation, but components used in circuits shown before are relatively complex and expensive.

1.3 Justification

Idea developed in this paper is of much simpler design, and less expensive. In applications where size of instrument has to be relatively small or application for which price of components is considered important, then our solution is more optimal than ones that are commonly used for comparing capacitance values of capacitors.

2 Methodology

When capacitor is connected in series with resistor it produces a system that responds to changes of input voltage with some latency and constant describing how fast or slow these elements respond is called time constant. Value of time constant τ is described by equation (11) such that R is the value of resistance involved and C for capacitance.

$$\tau = R \cdot C \tag{11}$$

We have decided that comparing time constant of a RC circuit is simplest way to compare capacitors electronically.

Comparing time constants Comparing time constants has a problem that sometimes resistor has different value of resistance with environmental changes. Hence, changes in time constant are not only because of the capacitor. We must add a potentiometer (variable resistors) instead of a resistor, solely for purpose of calibrating the circuit.

We have managed to find two effective ways to compare time constants.

1. Relaxation Oscillator – *frequency*
   Frequency of square wave relaxation oscillator depends on value of capacitor. Therefore, all what we need to do is compare frequencies of

two relaxation oscillators. Unfortunately, frequency comparison turns out to be fairly complex.

2. Astable Multivibrator - *duty cycle*
   In the circuit of astable oscillator there are two capacitors involved. Square wave with 0.5 duty cycle is produced when capacitors have the same value of capacitance. But duty cycle (and frequency) changes if one capacitor is larger than other. As a result of this fact, all we need to do is compare duty cycles of two signals.

Since it is relatively easier to compare duty cycles of two signals, we choose second method of comparing time constants.

2.1 Circuit design

General outline of a circuit is shown in Figure 5.

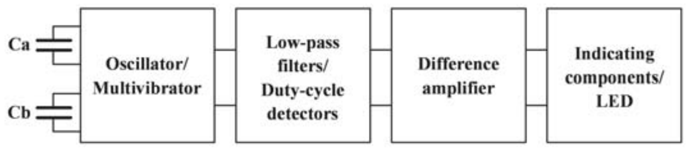

Figure 5: Outline of the comparator circuit

Capacitors Ca and Cb are to be compared, they are attached to the oscillator and capacitors control duty cycle of resulting signal. Purpose of low-pass filters is to find DC component of signals from oscillator. Difference amplifier is to amplify small differences between DC levels to be more distinguishable. LED components emit light to indicate whether first capacitor is larger than second or vice versa.

2.1.1 Astable Multivibrator

Astable Multivibrator is a two stage switching circuit in which the output of the first stage is fed to the input of the second stage and vice versa. This free running multivibrator generates square wave without any external triggering pulse. The circuit has two stable states and switches back and forth from one state to another, remaining in each state for a time depending upon the discharging of the capacitive circuit.[3]

Figure 6 shows the a astable multivibrator, square wave outputs V_1 and V_2 can be obtained from the collector point of Q1 or Q2, respectively.

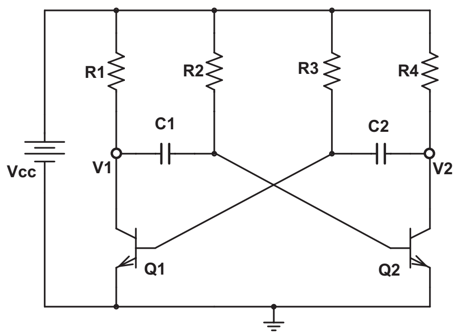

Figure 6: Astable multivibrator circuit

Imperfection in hardware will cause the first cycle, for our purpose we can ignore this detail and simply assume that currently Q1 is in state on.

Q1 holds the bottom of R1 (and the left side of C1) near ground ($0V$). The right side of C1 (and the base of Q2) is being charged by R2 from below ground to $0.7V$. R3 is pulling the base of Q1 up, but its base-emitter diode prevents the voltage from rising above $0.7V$. R4 is charging the right side of C2 up to the power supply voltage ($+V_{CC}$). Because R4 is less than R2, C2 charges faster than C1.

When the base of Q2 reaches $0.7V$, Q2 turns on, and the following positive feedback loop occurs.

Q2 abruptly pulls the right side of C2 down to near ($0V$). Because the voltage across a capacitor cannot suddenly change, this causes the left side of C2 to suddenly fall to almost $-V_{CC}$, well below $0V$. Q1 switches off due to the sudden disappearance of its base voltage. R1 and R2 work to pull both ends of C1 toward $+V_{CC}$, completing Q2's turn on. The process is stopped by the $B - E$ diode of Q2, which will not let the right side of C1 rise very far.

This now takes us to State 2, the mirror image of the initial state, where Q1 is switched off and Q2 is switched on. Then R1 rapidly pulls C1's left side toward $+V_{CC}$, while R3 more slowly pulls C2's left side toward $+0.7V$. When C2's left side reaches $0.7V$, the cycle repeats.

Voltage of capacitor C1 is

$$V_{C1} = V_{\text{FINAL}} + (V_{\text{INIT}} - V_{\text{FINAL}}) \cdot e^{-\frac{t}{RC}} \tag{12}$$

Voltage of $C1$ dischanges from $+V_{CC}$ to $-V_{CC}$ by conditions of circuit. The limiting point of it's discharge is approximately zero volts for which system changes state. Time required for voltage reach this zero is t_1. Applying this to equation (12) [4]

$$
\begin{aligned}
0V &= V_{CC} + (-V_{CC} - V_{CC}) \cdot e^{-\frac{t_1}{R_2 C_1}} \\
0.5 &= e^{-\frac{t_1}{R_2 C_1}} \\
-\ln 2 &= -\frac{t_1}{R_2 C_1}
\end{aligned}
$$

Therefore,

$$t_1 = R_2 C_1 \ln 2 \tag{13}$$

Similarly we can deduce that

$$t_2 = R_3 C_2 \ln 2 \tag{14}$$

Period of oscillations is $T = t_1 + t_2$ and frequency $f = \frac{1}{T}$. Outputs of V1 and V2 are shown in Figure 7.

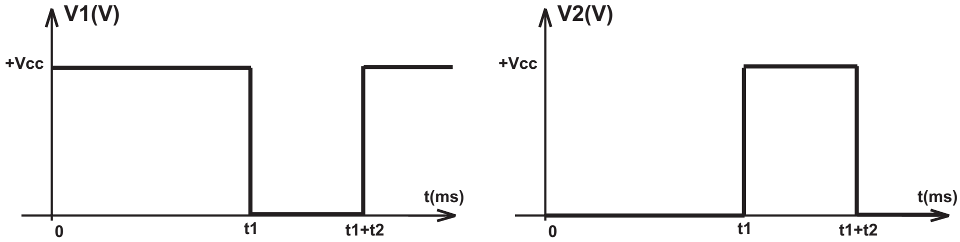

Figure 7: Output Waveforms of Astable Multivibrator

Real outputs are not ideal as the ones shown in Figure 7, but as long as we keep R_1 and R_4 much smaller than R_2 and R_3, we should not have a problem with assumptions that signals V_1 and V_2 are as depicted.

2.1.2 Range extension

In the circuit, values of compared capacitances cannot have a large range. For *small* values of capacitances frequency of output signals will be large and some oscillators are not capable of handling these frequencies. For *large* values of capacitors there are some issues which we will discuss later in this text.

Suppose that capacitance C can be attached to the oscillator in some range of for which oscillator will oscillate properly. That range is $(C_{LOW}) \leq C < (C_{LOW} + C_{HIGH})$. We must arrange a mapping[1] between all values of

[1] The term map or mapping is often used as a synonym for function.

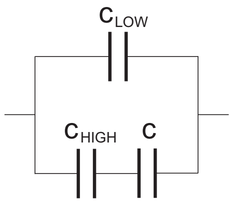

Figure 8: Range extending combination of capacitors.

capacitances from range $[0, \infty)$ to the range $[C_{LOW}, C_{LOW} + C_{HIGH})$. Such mapping must be strictly increasing for the sake of comparison[2]. Figure 8 shows an implementation of such mapping, and formula (15) describes relation between old and new value of capacitance. This technique we will call *range extension*, because we extend the range of values of capacitors.

$$C_{new} = C_{LOW} + \frac{C \cdot C_{HIGH}}{C + C_{HIGH}} \tag{15}$$

As we can see in the Figure 9, the range of $[0, \infty)$ was mapped onto $[2, 3)$. Range extension has a problem that for large values of capacitances, differences are much smaller; hence our comparator must have a large sensitivity in order to compare two very large capacitors. In the beginning the curve is very steep; which means that we will compare smaller values with much higher resolution.

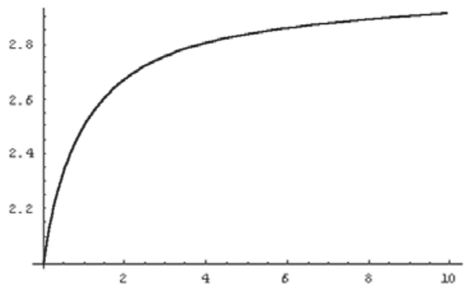

Figure 9: Mapping between C and C_{new}; for values of C_{LOW}=2, C_{HIGH}=1

2.2 Circuit schema

Based on general scheme for our design, we implement individual parts of a design and incorporate components as in Figure 10. Low pass filters consist

[2]If $a < b$ then $f(a) < f(b)$ only when function $f(x)$ is strictly increasing.

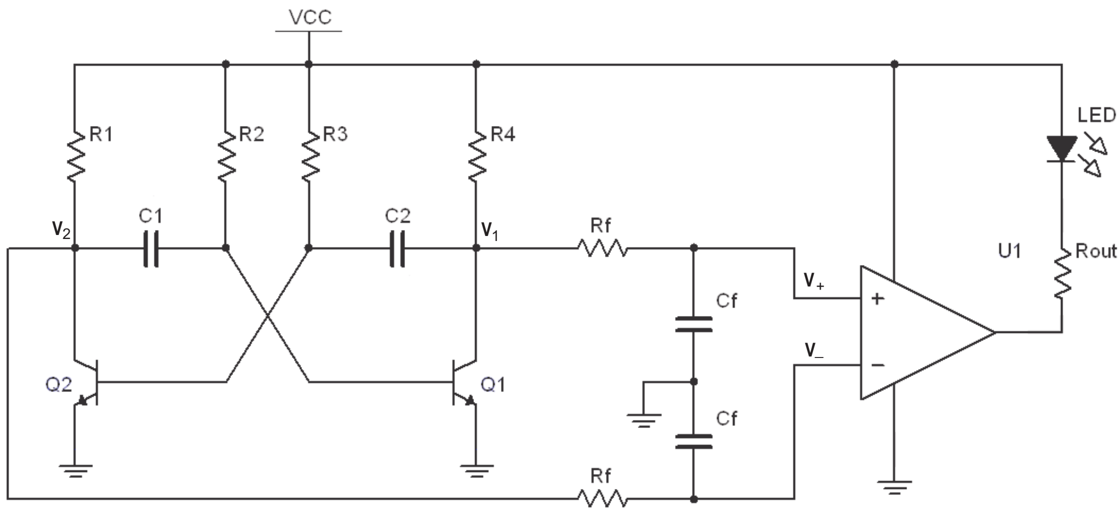

Figure 10: Circuit schema of capacitances comparator

of Rf and Cf, DC levels of V1 and V2 are fed into a OPAMP which compares the DC levels and gives the output either Vcc or ground.[5]

When $C_1$is larger than C_2, output of OPAMP will be ground and voltage will appear across R_{out} and LED, hence LED will be ON. On the other hand when $C_2$is larger than C_1, output od OPAMP will be equal to Vcc, therefore LED will be OFF.

2.2.1 DC levels

After we have signals as in Figure 7, low pass filter will give an average of those signals as it's output. For signals V_1 and V_2, DC levels are DC_1 and DC_2 respectively.

$$DC_1 = Vcc\frac{t_1}{t_1 + t_2} \tag{16}$$

$$DC_2 = Vcc\frac{t_2}{t_1 + t_2} \tag{17}$$

3 Results

In this section we will be discussing the circuit's performance. Primary concern of our design is sensitivity. Low Pass filter will allow some high frequencies to pass through [7]. Hence, output of low pass filter will fluctuate around the DC level as show in Figure 11. This fluctuation will interfere with our readings.

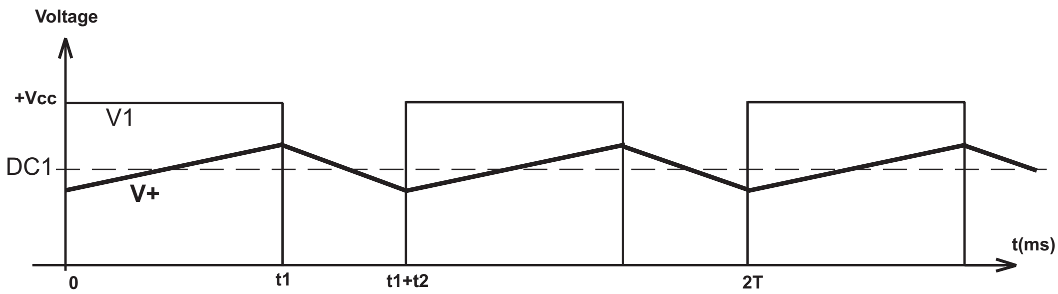

Figure 11: Fluctuations of DC level due to imperfections of low pass filter.

If difference between DC levels of V1 and V2 is not large enough, OPAMP will cause the LED to blink, i.e. change states between ON and OFF with frequency same as Multivibrator, blinking of LED means that capacitors are closely similar. Measurable difference or larger difference of capacitors will make a definite result (ON or OFF). Any difference of capacitors that is less than measureable will cause the blinking and uncertanty.

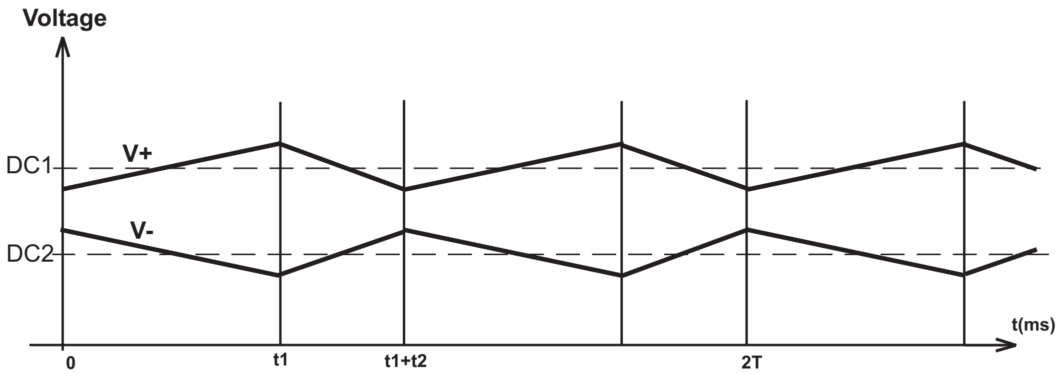

Figure 12: Comparison between two inputs to the OPAMP.

Figure 12 shows the inputs to the OPAMP, our concern is to keep them separate. At times $t = 0$ and $t = k(t_1 + t_2) = k \cdot T$ inputs are closest to each other and we must find a relation that will provide the condition in equation (18).

$$V_+(0) \geq V_-(0) \tag{18}$$

Low pass filter will react as a RC circuit with time constant $\tau_f = R_f \cdot C_f$. When system stabilises we have the obvious $V_+(0) = V_+(t_1+t_2)$ and $V_-(0) = V_-(t_1 + t_2)$.

Considering the charging and discharging of C_f we conclude two relations

$$V_+(t_1) = V_+(0) + (Vcc - V_+(0))\left(1 - e^{\frac{-t_1}{\tau_f}}\right) \tag{19}$$

$$V_+(0) = V_+(t_1 + t_2) = V_+(t_1)\left(e^{\frac{-t_2}{\tau_f}}\right) \tag{20}$$

Equation (19) represents that after time t_1, voltage will change to new value due to charging up the capacitor, and equation (20) is opposite process, during t_2 voltage changes back to original value $V_+(t_1 + t_2) = V_+(0)$.

We can solve the system of equations (19) and (20) to get solution for $V_+(0)$ as in the equation (21).

$$V_+(0) = Vcc\frac{\left(e^{\frac{t_1}{\tau_f}} - 1\right)}{\left(e^{\frac{t_1+t_2}{\tau_f}} - 1\right)} \tag{21}$$

Similarly we can apply same principles for $V_-(0)$ and $V_-(t_1)$ to get the solution for $V_-(0)$ as in the equation (22).

$$V_-(0) = Vcc \cdot e^{\frac{t_1}{\tau_f}}\frac{\left(e^{\frac{t_2}{\tau_f}} - 1\right)}{\left(e^{\frac{t_1+t_2}{\tau_f}} - 1\right)} \tag{22}$$

We supstitute with equations (13),(14), and consider that $C_1 = C + \Delta C$ and $C_2 = C$, furthermore we might assume that $R_2 = R_3 = R$. Equating $V_+(0) = V_-(0)$ based of (18) gives us the following result (23).

$$1 + 2^{\frac{R(2C+\Delta C)}{\tau_f}} = 2^{1+\frac{R(C+\Delta C)}{\tau_f}} \tag{23}$$

ΔC is the *minimal measurable difference* between two capacitors of sizes C and $C + \Delta C$. Solving the equation (23) for ΔC yields to (24).

$$\Delta C = -2C - \frac{T \ln\left(2^{1-\frac{CR}{\tau_f}} - 1\right)}{R \ln 2} \tag{24}$$

Using Taylor series expansion[6] of equation (24), we can get much simpler result (25).

$$\Delta C = \frac{R \ln 2}{R_f C_f} C^2 \tag{25}$$

This amazingly simple, yet useful, formula can give us a minimal value of difference between two capacitors that are distinguishable by our design. Figure 13 shows a plot of equation (25) for some values of parameters.

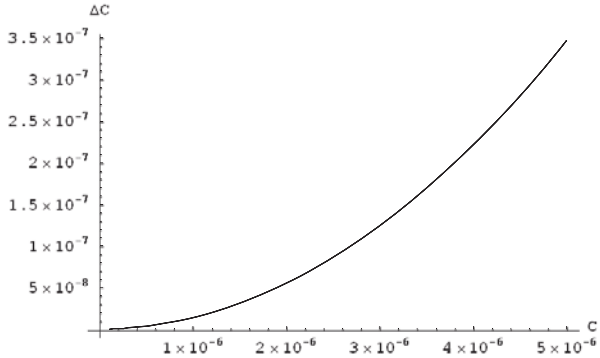

Figure 13: Plot of equation (25) for $R = 10k\Omega$, $R_f = 10k\Omega$, $C_f = 50\mu F$

To give example of previous concepts consider that parameters of circuit are $R = 10k$, $R_f = 10k$, $C_f = 50\mu F$. If the first capacitor has value $C = 1\mu F$, then other capacitor can be with value $C + \Delta C = 1.014\mu F$, and they will be distinguished. Any difference lower than this value, will cause LED to blink, indicating that capacitors are equal.

In practical experiments we found that the behaviour of circuit shows correspondance with equation (25).

It is also important to note that it is not practical to choose $R_f C_f$ too large, because response of circuit to the changes of capacitors will not be fast enough. For example in Figure 14 we have shown voltages V_+ and V_-, and in this example it took arround $0.4s$ to respond to the changes.

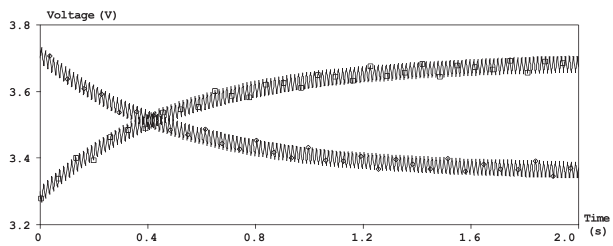

Figure 14: Circuit does not repond to the changes immediately, plot of $V_+$and V_-.

From Figure 14, we observe that on average signal behaves similar to a capacitor charge or discharge. It would be difficult to obtain the actual formula governing this behaviour, but we can give approximate response time T_R.

$$T_R = R_f C_f \left(\frac{\Delta C}{C}\right)^{-1/3} \tag{26}$$

The formula (26) is solely based on numerical evidence based on simulation using PSPICE software. Respose time given with equation (26) is the worst possible case, actual value of response time may be half of that or less. Figure 15 shows test points and approximating curve.

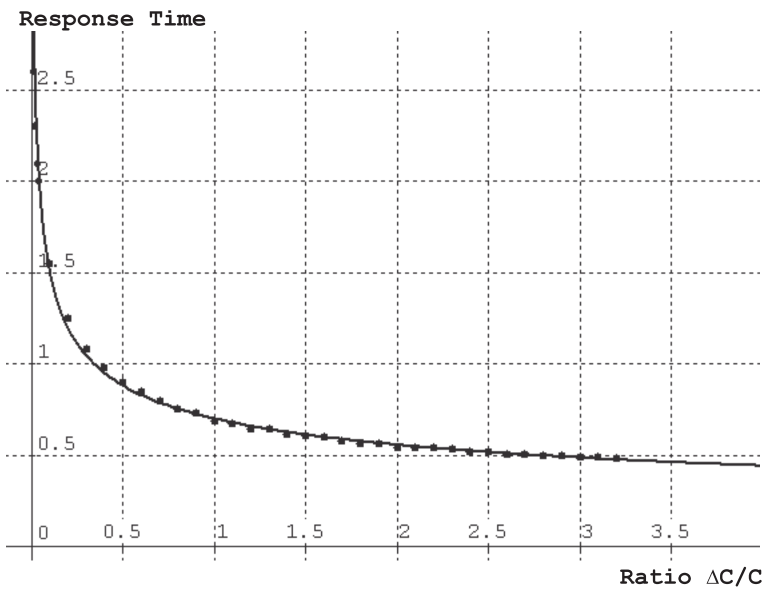

Figure 15: Plot of ratio $\frac{\Delta C}{C}$; against time in seconds; for $R_f C_f = 0.5$

3.1 Discussion

For our circuit to have good performance it must have high sensitivity and fast response, unfortunatelly it is difficult to achieve both simultaneously. Examine equations (26) and (25), increasing $R_f C_f$ will cause ΔC to decrease but T_R will raise. And if we decrease $R_f C_f$ then ΔC will increase, but circuit will be more responsive because T_R will drop. It is difficult to decide which parameter is more important ΔC (sencitivity) or T_R (response time).

We must make compromises leaving a response time a bit larger to make it more sensitive for capacitor differences, or vice versa. A way to improve this is to use some high-order active low-pass filter instead of passive filter, this will increase complexity but reduce ΔC and T_R, for some applications this might be necessary.

Actually, in this circuit it is possible to decrease both ΔC or T_R simultatenously but at a cost of consuming more power. Observe that in equation (25) we also have extra parameter $R = R_2 = R_3$. Decreasing a $R_f C_f$ will cause increase in ΔC but if we decrease $R_2 = R_3$ simultaneously, we will compensate such that ΔC remains small. Basically we can have a circuit that is fast and responsive simultaneously, but another problem arrises. Resistors $R_1 = R_4$ should be at least couple of times smaller that $R_2 = R_3$ (refer to the Figure 6). Voltage Vcc will be directly applied to $R_1 = R_4$ and power consumption of these resistors is determined by equation (27).

$$P = \frac{Vcc^2}{R_1} \tag{27}$$

For example, setting $Vcc = 7V$ and let's say that $R_1 = 100\Omega$, then power consumed by $R_1 = R_4$ is $P = 490mW$. High power consumption will cause heating of components and extreme heating might cause some other undesireable effects.

As usual in engineering, we have a couple of parameters to vary. Different values of parameters will develop into a different solutions. But each solution will have some benefits and drawbacks.

4 Conclusion

An electronic circuit has been presented which compares values of two capacitors. The component requirement is minimum and of relatively low cost. Besides the simple configuration, it features high sensitivity over wide range of capacitances. Limitations of this circuit have been discussed. If difference between two capacitors is lower than given value ΔC circuit will not make a definite decision, and those capacitors are considered equal. Response time of circuit is relatively large which makes it considerably difficult to use in high speed applications. In practical applications, this circuit can be used to monitor some physical quantity by connecting the appropriate capacitive transducers to be compared [8].

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